PHYSICS 534

EXERCISE-23 **Law of Gravitation**

BARKLA

Charles Barkla was awarded the Nobel prize for physics in 1917 for his X-ray spectra of the elements.

The weight of an object is the gravitational attraction between the object and planet earth. The law of universal gravitation was discovered by Isaac Newton and was first announced by him in 1886. It says:

> ***Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.***

Mathematically, the law of universal gravitation is written: $F_G = G\frac{m_1m_2}{r^2}$

where: F_G = the gravitational force, in newtons (N)
G = gravitational constant = 6.67×10^{-11} ($N{\cdot}m^2/kg^2$)
m_1 = mass of first object, in kilograms (kg)
m_2 = mass of second object, in kilograms (kg)
r = the distance between objects, in metres (m)

- ***Note***: Newton's law of universal gravitation is considered by most scientists to be the greatest scientific discovery of all time.

---

1. Two subway trains, each of mass 4.0×10^5 kg, are located with their centers of mass 8 m apart. What gravitational force exists between them? [0.17 N]

$$F_G = \frac{GM_1M_2}{r^2} = \frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(4.0\times10^5\ kg)(4.0\times10^5\ kg)}{(8\ m)^2}$$

$$= 0.17\ N$$

2. An 80 kg boy and a 60 kg girl are 2 meters apart. What is the gravitational force of attraction between them? [8×10^{-8} N]

$$F_G = \frac{GM_1M_2}{r^2} = \frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(80\ kg)(60\ kg)}{(2\ m)^2}$$

$$= 8\times10^{-8}\ N$$

3. What is the gravitational force of attraction between two 1 000 kg automobiles which are 5 m apart? [2.67×10^{-6} N]

$$F_G = \frac{GM_1M_2}{r^2} = \frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(1000\ kg)(1000\ kg)}{(5\ m)^2}$$

$$= 2.67\times10^{-6}\ N$$

4. What is the gravitational force of attraction between two 1 000 kg cars which are 50 m apart? [2.67 x 10^{-8} N]

$$F_G = \frac{GM_1M_2}{r^2} = \frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(1000\ kg)(1000\ kg)}{(50\ m)^2}$$
$$= 2.67\times10^{-8}\ N$$

5. Two trucks are parked next to each other. Their centers of gravity are 10 meters apart. One truck weighs 8 x 10^5 N. The other weighs 6 x 10^4 N. What gravitational force exists between them? (Assume g = 10 m/s^2) [3.2 x 10^{-4} N]

Note: Convert the weights to masses.

$$F_G = \frac{GM_1M_2}{r^2} = \frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(8\times10^4\ kg)(6\times10^3\ kg)}{(10\ m)^2}$$
$$= 3.2\times10^{-4}\ N$$

6. Two space capsules, each of mass 2 000 kg, are put into orbit 30 m apart. What gravitational force exists between them? [3.0 x 10^{-7} N]

$$F_G = \frac{GM_1M_2}{r^2} = \frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(2000\ kg)(2000\ kg)}{(30\ m)^2}$$
$$= 3.0\times10^{-7}\ N$$

7. In a hydrogen atom, an electron and a proton are 1.0 x 10^{-10} m apart. What is the gravitational force of attraction between them? [1.01 x 10^{-47} N]

- ***Note***: Mass of electron = 9.11 x 10^{-31} kg
  Mass of proton = 1.67 x 10^{-27} kg

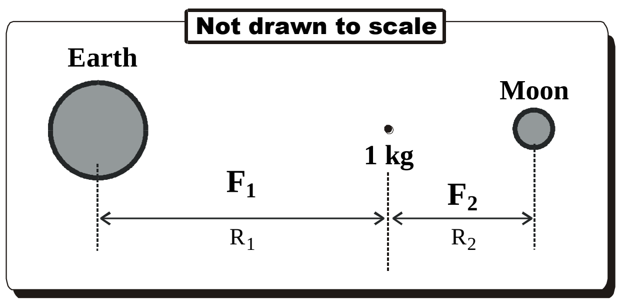
$$F_G = \frac{GM_1M_2}{r^2} = \frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(9.11\times10^{-31}\ kg)(1.67\times10^{-27}\ kg)}{(1.0\times10^{-10}\ m)^2}$$
$$= 1.02\times10^{-47}\ N$$

8. The mass of the earth is 5.98 x 10^{24} kg. The mass of the moon is 7.35 x 10^{22} kg. Knowing that the distance between the earth and the moon is 3.85 x 10^{5} km, calculate the force of attraction between the earth and the moon. [1.98 x 10^{20} N]

$$F_G = \frac{GM_E M_M}{R^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(7.35\times10^{22}\ \text{kg})}{(3.85\times10^{8}\ \text{m})^2}$$

$$= 1.98\times10^{20}\ \text{N}$$

9. Calculate at what point from the earth do the gravitational fields of the Earth and the moon cancel each other. [3.47 x 10^{8} m]



M_E = Mass of the earth $\quad M_M$ = Mass of the moon $\quad M = 1$ kg mass

$$F_1 = \frac{GM_E M}{R_1^2} \qquad F_2 = \frac{GM_M M}{R_2^2}$$

But $F_1 = F_2$

$$\therefore \quad \frac{GM_E M}{R_1^2} = \frac{GM_M M}{R_2^2} \quad \text{or} \quad \frac{M_E}{R_1^2} = \frac{M_M}{R_2^2} \quad \text{or} \quad \frac{5.98\times10^{24}\ \text{kg}}{R_1^2} = \frac{7.35\times10^{22}\ \text{kg}}{R_2^2}$$

$$\therefore \frac{R_1^2}{R_2^2} = \frac{5.98\times10^{24}}{7.35\times10^{22}} = 81.36 \quad \text{or} \quad \frac{R_1}{R_2} = 9 \quad \text{or} \quad R_1 = 9R_2$$

Since $R_1 + R_2 = 3.85\times10^{8}$ m

$$\therefore \quad 9R_2 + R_2 = 3.85\times10^{8}\ \text{m} \quad \text{or} \quad 10R_2 = 3.85\times10^{8}\ \text{m} \quad \text{or} \quad R_2 = \frac{3.85\times10^{8}\ \text{m}}{10} = 3.85\times10^{7}\ \text{m}$$

and $R_1 = 3.85\times10^{8}\ \text{m} - 3.85\times10^{7}\ \text{m} = 3.465\times10^{8}\ \text{m} = 3.47\times10^{8}\ \text{m}$

10. An astronaut weighs 600 N here on Earth. Calculate his weight for each case below.
➲ **Hint: Use ratio and proportion.**

a) If the astronaut were standing on a step-ladder equal in height to the radius of the earth (6400 km), how much would he weigh? [150 N]

The relation between force and distance is inverse

Since the reciprocal of 2 *squared* $= (1/2)^2 = 1/4$

$\therefore \frac{1}{4}(600\ N) = 150\ N$

b) If he were standing on a planet the same mass as the earth but half its radius, how much would he weigh? [2 400 N]

The relation between force and distance is inverse.

Since the reciprocal of $1/2$ *squared* $= (2)^2 = 4$

$\therefore 4(600\ N) = 2400\ N$

c) If he were standing on a planet twice the radius of the earth but of the same mass, how much would he weigh? [150 N]

The relation between force and distance is inverse.

Since the reciprocal of 2 *squared* $= (1/2)^2 = 1/4$

$\therefore \frac{1}{4}(600\ N) = 150\ N$

d) If he were standing on a planet the same radius as the earth but twice its mass, how much would he weigh? [1 200 N]

The relation between force and mass is direct.

$\therefore 2(600\ N) = 1200\ N$

e) If he were standing on a planet twice the radius of the earth and four times its mass, how much would he weigh? [600 N]

The relation between force and distance is inverse while the relation between force and mass is direct.

Thus, the effect of a larger distance ($F_G = 1/4$) is cancelled by the effect of a larger mass ($F_G = 4$).

$\therefore \frac{1}{4}(4)(600\ N) = 600\ N$